

Bulk Scale Factor at Very Early Universe

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Abstract

In this paper we propose a higher dimensional Cosmology based on FRW model and brane-world scenario. We consider the warp factor in the brane-world scenario as a scale factor in 5-dimensional generalized FRW metric, which is called as *bulk scale factor*, and obtain the evolution of it with space-like and time-like extra dimensions. It is then showed that, additional space-like dimensions can produce exponentially bulk scale factor under repulsive strong gravitational force in the empty universe at a very early stage.

Keywords: FRW Model; Scale Factor; de-Sitter space-time; Extra Dimensions

1 Introduction and Motivation

In 1917, a year after Einstein introduced general relativity, he derived a static cosmological model which was closed [1]. His motivation in driving this model was the strong believe that the universe is static. For this reason, he introduced cosmological constant in the field equation which is,

$$\Lambda_0 = 8\pi G\rho_0 \quad (1)$$

Where G is Newton's constant of gravitation and ρ_0 is density of the whole universe.

In the same year, W. de-Sitter obtained another solution to the modified Einstein's field equation [2]. The de-Sitter solution reads as equations [3]

$$ds^2 = dt^2 - a^2(t)[dx_1^2 + dx_2^2 + dx_3^2] = dt^2 - e^{2Ht}[dx_1^2 + dx_2^2 + dx_3^2] \quad (2)$$

Where 1,2,3 indices refer to the three spatial dimensions and $a(t)$ is scala factor and $H = \sqrt{\frac{\Lambda}{3}}$. The de-Sitter metric is a solution to homogenous Einstein's equation, *i.e.* for empty universe and in the past two decades, observational data have shown that our universe might be in de-Sitter phase.

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The most successful model, having wondrous predictive power for cosmology, was obtained by Friedmann in 1922 [4], as well as by Robertson and Walker in 1935 and 1936, respectively [5, 6], which is called as Friedmann-Robertson-Walker (FRW) model. This expanding and centrally symmetric homogeneous model obey the cosmological principle. The successfulness of this model is revealed by the fact that the big-bang model is based on this model [7, 8].

After Hubble's observation, in 1929, Einstein realized that he committed a mistake by introducing the cosmological constant. Due to this epoch-making observation, cosmologists were prompted to think for expanding models of the universe more seriously, though de-Sitter and Friedmann had already derived non-static models by this time. So, it was natural to think that if the present universe is so large, it would have been very small and dense in the extreme past. In 1940, George Gamow addressed to this question and proposed that, in the beginning of its evolution, universe was like an extremely hot ball [9]. He called this ball as primeval atom and this model is popularly known as the big-bang model.

On the other hand, the idea that our universe may consist of more dimensions than the usual 4 dimensional space-time, was first considered by T. Caluza in the 1919 [10, 11, 12]. His aim was to unify gravity and electromagnetism. Later in 1999, the 5D (5-dimensional) warped geometry theory, which is a brane-world theory, developed by Lisa Randall and Ramam Sundrum, while trying to solve the hierarchy problem of the Standard Model, which is called RS model [13, 14]. They considered one extra "non-factorizable" dimension and the metric was found for RS model to be given by [15, 16, 17]

$$ds^2 = g_{\mu\nu}a^2(y)dx^\mu dx^\nu - dy^2 = g_{\mu\nu}e^{-2ky}dx^\mu dx^\nu - dy^2 \quad (3)$$

Where Greek indices $\mu, \nu = (0, 1, 2, 3)$ are refer to the usual four observable dimensions, and y signifies the coordinate on the additional dimension of length r and $a(y)$ is called the warp factor and k is of order of the Plank mass. Here it is assumed to have two branes. One at $y = 0$ called Planck-brane (where gravity is relatively a strong force) and another at $y = \pi r$ called the TeV-brane (where gravity is relatively a weak force). In that case, by warping any lagrangian mass parameter which is naturally $\approx M_{PL}$ (Planck mass), will appear to us in the 4D space-time on the TeV-brane to be $\approx TeV$ (Tera electron-volt). For example, lets see how this works for the Higgs field on the TeV-brane. First, we right down the action for the Higgs field in 5D

$$S = \int d^4x dy \sqrt{-g} [g^{\mu\nu} \partial_\mu \hat{H}^\dagger \partial_\nu \hat{H} - \lambda(\hat{H}^2 - \nu_0^2)^2] \delta(y - \pi r) \quad (4)$$

Where ν_0 is the vacuum expectation value of the Higgs field and it is of order of the Planck mass, and \hat{H} is Higgs field operator. The integration over extra dimension is easy to calculate

$$S = \int d^4x [e^{-2kr\pi} \partial_\mu \hat{H}^\dagger \partial^\mu \hat{H} - e^{-4kr\pi} \lambda(\hat{H}^2 - \nu_0^2)^2] \quad (5)$$

If we re-scale the Higgs field, $\hat{H} \rightarrow e^{kr\pi} \hat{H}$, this gives:

$$S = \int d^4x [\partial_\mu \hat{H}^\dagger \partial^\mu \hat{H} - \lambda(\hat{H}^2 - \nu_0^2 e^{-2kr\pi})^2] \quad (6)$$

Thus the Higgs field is still seen as a Higgs field in four dimensional space-time, but the vacuum expectation value is now

$$\nu^2 = \nu_0^2 e^{-2kr\pi} \quad (7)$$

Which is at the TeV-scale. Thus by considering warped extra dimensions one is able to solve the hierarchy problem. This means that transition from 4D world to 5D world, exponentially shrink sizes and grows mass and energy [18, 19, 20].

The possibility of the existence of extra dimensions has opened up new and exciting avenues of research in quantum gravity and quantum cosmology (for instance see recently works [21, 22, 23, 24, 25]). Since a higher dimensional world has more energy (the range of Planck-scale) relative to 4D world (the range of TeV-scale), in this paper, similar to Gamow idea, we suggest higher dimensional world as primeval atom. In contrast to warped geometry model, which makes transition from 4D world to the 5D world, we go from 5D world to the 4D world. By considering the symmetry, an expansion in size in our scenario is expected. Indeed, we intend to propose a higher dimensional Cosmology based on FRW model and brane-world scenario. For this purpose, We consider generalized form of FRW metric in 5D space-time and by solving of the Einstein equation for this 5D space-time, we obtain the evolution of the bulk scale factor. Thus, the outline of paper is as follows: In section 2, the scale factor in 4D de-Sitter space-time is recalled briefly. In section 3, we consider the evolution of bulk scale factor in 5D space-time with space-like and time-like extra dimensions. Some conclusions are given in final section.

2 Scale factor in 4D de-Sitter space-time

The standard FRW space-time is[7]

$$ds^2 = dt^2 - a^2(t)[dx_1^2 + dx_2^2 + dx_3^2]. \quad (8)$$

Where 1,2,3 indices refer to the three spatial dimensions. In this case, for empty universe, $T_{\mu\nu} = 0$, and cosmological constant, $\Lambda \neq 0$, which is the source of gravitation in place of a massive object, the Enistein's field equation reads

$$R_{\mu\nu} = \Lambda g_{\mu\nu}. \quad (9)$$

Then we have:

$$-\frac{3\ddot{a}}{a} = \Lambda. \quad (10)$$

and

$$a\ddot{a} + 2\dot{a}^2 = \Lambda(-a^2). \quad (11)$$

Then from (10) and (11) we have:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\Lambda}{3}.$$

Which integrates to:

$$a(t) = a_0 e^{Ht} = a_0 e^{t\sqrt{\frac{\Lambda}{3}}}. \quad (12)$$

Where a_0 is the scale factor at the end of period inflation in 4D de-Sitter space-time. So, non-static form of de-Sitter space-time with exponentially scale factor is obtained as

$$ds^2 = dt^2 - a_0^2 e^{2Ht}[dx_1^2 + dx_2^2 + dx_3^2]. \quad (13)$$

3 Evolution of Bulk scale factor

By considering the standard FRW metric given by (8) and the metric (3), generalized form RS brane-world scenario [15], we suggest the generalized form of 5D FRW metric as

$$ds^2 = g_{ab}dx^a dx^b = \eta_{\mu\nu}a^2(y)dx^\mu dx^\nu + r^2 dy^2, \quad (14)$$

Where

$$g_{ab} = \text{diag}(a^2(y), -a^2(y), -a^2(y), -a^2(y), r^2). \quad (15)$$

Which $a, b = (0, 1, 2, 3, 4)$ and Greek indices $\mu, \nu = (0, 1, 2, 3)$, refer to the four observable dimensions, and y signifies the coordinate on the additional dimension of length r , and scale factor $\tilde{a} = a(y)$, is a *bulk scale factor*, which only respect to the extra dimension coordinate y . For space-like extra dimensions (**SLED**), $r^2 = -1$ and for time-like extra dimensions (**TLED**), $r^2 = +1$ [26, 27, 28].

Similar to section 2, for empty universe which $T_{ab} = 0$, and bulk cosmological constant, $\tilde{\Lambda} \neq 0$, the Enstein's field equation reads as

$$R_{ab} = \tilde{\Lambda}g_{ab}. \quad (16)$$

Then for **TLED** we have:

$$-3\dot{\tilde{a}}^2 - \tilde{a}\ddot{\tilde{a}} = \tilde{\Lambda}\tilde{a}^2, \quad (17)$$

Which $ab = (00, 11, 22, 33)$, and for $ab = 44$, we have

$$\frac{4\ddot{\tilde{a}}}{\tilde{a}} = \tilde{\Lambda}. \quad (18)$$

Where dot denotes derivative with respect to y as a **TLED**.

From Eq.(17) together with Eq.(18), we have:

$$\left(\frac{\dot{\tilde{a}}}{\tilde{a}}\right)^2 = \tilde{H}^2 = \frac{-5\tilde{\Lambda}}{12}$$

So we obtain,

$$\tilde{a} = \tilde{a}_0 e^{\tilde{H}y}. \quad (19)$$

Where \tilde{H} is 5D Hubble parameter, and \tilde{a}_0 is the bulk scale factor at the end of its expansion in 5D space-time. In this case, $\tilde{\Lambda} < 0$, and we have an expanding space-time with harmonically scale factor (19) as:

$$ds^2 = \tilde{a}_0^2 e^{2\tilde{H}y} [dt^2 - (dx_1^2 + dx_2^2 + dx_3^2)] + dy^2. \quad (20)$$

On the other hand, the Enstein's field equation, $R_{ab} = \tilde{\Lambda}g_{ab}$, for **SLED** yields to:

$$3\tilde{a}'^2 + \tilde{a}\tilde{a}'' = \tilde{\Lambda}\tilde{a}^2. \quad (21)$$

where $ab = (00, 11, 22, 33)$, and for $ab = 44$ we have:

$$\frac{-4\tilde{a}''}{\tilde{a}} = \tilde{\Lambda}(-1). \quad (22)$$

Where prime denotes derivative with respect to y as a **SLED**.

Then from (21) and (22), we have:

$$\left(\frac{\tilde{a}'}{\tilde{a}}\right)^2 = \tilde{H}^2 = \frac{\tilde{\Lambda}}{4}$$

Which integrates to:

$$\tilde{a} = \tilde{a}_0 e^{\tilde{H}y} = \tilde{a}_0 e^{y\sqrt{\frac{\tilde{\Lambda}}{4}}} \quad (23)$$

Where \tilde{H} is 5D Hubble parameter, and \tilde{a}_0 is the bulk scale factor at the end of its expansion in 5D space-time. In this case, $\tilde{\Lambda} > 0$, and we have an expanding space-time with exponentially scale factor (23) as:

$$ds^2 = \tilde{a}_0^2 e^{2\tilde{H}y} [dt^2 - (dx_1^2 + dx_2^2 + dx_3^2)] - dy^2. \quad (24)$$

Though, in the brane picture, the electromagnetism and the weak and strong nuclear forces are localized on the TeV-brane, but gravity has no such constraint and so much of its attractive power "leaks" into the bulk and the force of gravity should appear significantly stronger on small scales. So, the form of expansion (24), can be as a consequent of repulsive of this strong gravitational force in the very early universe with extra dimensions.

4 Discussions and Conclusions

We considered the generalized form of FRW metric similar to generalized RS brane-world scenario, which admits both **SLED** and **TLED**. We solved the Einstein equation for an empty 4D and 5D space-time and demonstrated that the evolution of the bulk scale factor in 5D space-time for **SLED** case (24), is exponential form similar to scale factor in 4D space-time (13). Besides, the following results and points can be obtained from this work:

- In this scenario, due attention to hierarchy problem and brane-world Cosmology, it seems more logical that, the 5D very early universe to have further energy in comparison with the 4D early universe.

- In the higher dimensional very early universe, incorporation of hidden extra dimensions and the visible 4D space-time, the unification of gravitational force and gauge forces seems to be more possible.

- In this scenario, for TLED we have an harmonically bulk scale factor (20) and the strong gravitational force is likely harmonic (similar to non-static form of anti de-Sitter 5D space-time).

- In this scenario, for SLED we have an exponentially bulk scale factor (24) and the strong gravitational force is likely repulsive (similar to non-static form of de-Sitter 5D space-time). On the other hand, in the brane-world model, by warping any lagrangian mass, shrink size and the strong gravitational force is likely attractive. Consequently, in the bulk and in the Planck scale of energy, gravitation can be attractive as well as repulsive. Thus, the quantum effects of gravitation are of great importance [24].

- This mechanism of expansion emanate of transition from unstable and dense state of 5D world (where is situate on the exited mode of Kaluza-Klein model and under effects of strong gravitational force) to the 4D world (where is under effects of weak gravitational force with lesser energy) [29].

These highlight points, will motivate us to suggest another physics for very early universe in the higher dimensional Cosmology.

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